

**MODE IDENTIFICATION OF OSCILLATIONS
OF DELTA SCUTI TYPE STARS USING HIGH
TEMPORAL RESOLUTION KEPLER DATA**

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ABSTRACT

MODE IDENTIFICATION OF OSCILLATIONS OF DELTA SCUTI TYPE STARS USING HIGH TEMPORAL RESOLUTION KEPLER DATA

Asteroseismology is the Astrophysical science that studies the oscillations in the light curves of variable stars to understand the internal stellar structure of variable stars. This research aimed to construct light curves from the Kepler Mission data and calculate basic internal properties of the selected stars by analysing the modes of the stars. Accordingly, Photometric data of Kepler mission are taken to conduct an asteroseismic investigation on three chosen Delta Scuti type variable stars. The Kepler Mission was launched to search the Milky Way galaxy to discover the Earth-size and smaller planets, which can contain habitable zones. This mission took hundreds of thousands of data from all the types of stars. The names given from the Kepler catalogue for the selected Delta Scuti stars are KIC 4048494, KIC 4077032 and KIC 8623953. To conduct the research on the star KIC 4048494, two months long short cadence data was taken; for the star KIC 4077032, one month long short cadence data was taken; for the star KIC 8623953, three months long short cadence data was taken. The data were taken from the KASOC database and further corrections of eliminating photometric jumps was performed. These high resolution data allow to construct power spectra that can resolve frequencies up to $1.16 \mu\text{Hz}$.

Light curve analysis for each of the variable stars is conducted and presented. The pulsation frequency modes were determined using the Period04 software which is a C++/Java based program that dedicated to statistical analysis of large astronomical time series. The rotations and pulsations of Delta Scuti stars are very complex. There are numerous unknown oscillation types still exists for the Delta Scuti type variable stars. The main frequency identification and frequency combination determination was done and the pulsation constant (Q) for each and every frequency was calculated. The radial fundamental mode and the 1st overtone of each star was determined by observing their period ratios and considering their amplitude variations. For the star KIC 4048494, the fundamental radial mode was $127.992 \mu\text{Hz}$ and the first overtone was $166.244 \mu\text{Hz}$. The fundamental radial mode for the star KIC 4077032 was $73.048 \mu\text{Hz}$ and the first overtone was $93.025 \mu\text{Hz}$. The fundamental radial frequency of the star KIC 8623953 was $315.473 \mu\text{Hz}$.

Keywords: Delta Scuti stars, Photometry, Oscillation modes, Stellar pulsations.

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LIST OF ABBREVIATIONS

DSCT	Delta Scuti
KASOC	Kepler Asteroseismic Science Operations Center

CHAPTER 1

INTRODUCTION

1.1. Variable Stars

Astrophysics is the science that dedicated to understanding the physics and chemical phenomena that deals with celestial objects such as planets, stars, nebulas, galaxies and other celestial objects. Studying stellar physics is a significant part of astrophysics. Stars are the building blocks of the entire universe and all the visible energy earth receive are comes from the stars. Basically, they are giant balls of gas that in the balance of the force of gravity trying to crush all the mass into its core and the outward force of nuclear burning of the core which trying to blow it apart. If one of these forces wins the fight against each other the loses its life, either exploding out into space, a phenomenon called a supernova or crushed into a singularity which forms a black hole. Stars form pairs and clusters and billions of them can form galaxies and even more bigger structures. Further, the stars are very significant part in the formation of planets, asteroids and comets and hosting them as well. Specifically, the existence of biological life in the universe thoroughly depends on the stars.

The variable stars are leading counterpart of determining information of stellar astrophysics. Stars that change its brightness with time is called variable stars. If its apparent magnitude is altered in any way from our perspective on Earth the star is considered as a variable star. These changes can range from one-thousandth of a magnitude to 20 in magnitudes and they can occur over years or just fractions of a second. Over 100,000 variable stars have been identified and cataloged, with many more suspected variables. The Sun is an example of a variable star. The energy output of the sun varies by approximately 0.1%, or one-thousandth of its magnitude in an 11-year solar cycle [01]. There are mainly two types of variable stars; Intrinsic variables and extrinsic variables (see Fig 1.1).

1.1.1. Extrinsic Variables

The extrinsic variables are the stars whose brightness changes due to their external properties like being eclipsed by stellar rotation or by another star or planet. The variance of the total outward energy of the star is not the primary reason for the variability of the star. There are three main types of extrinsic variables, Eclipsing variables, Rotating variables and microlensing variables. When the orbital plane of a star coincides with our line of sight with another companion of the star, it is called an eclipsing variable. When a star has spots on its surface and with its rotation, the star can appear brighter at one time and dimmer otherwise. This is called rotating variables. Furthermore, when in a binary star system a pair of stars is tidally locked in close range and if one star is highly heated, part of the radiation reflects on the other star. When it rotates, the reflecting surface area changes and causes to change the brightness of the star. When an object which can act as a gravitational lens passes in front of a star it can change the brightness of the star and this type of stars are called microlensing variables [02].

1.1.2. Intrinsic Variables

The intrinsic variables are the stars which have changing luminosity physically because of pulsations, eruptions or through swelling and shrinking phenomena. There are several types of intrinsic stars based on their physical behavior. They are pulsative variables, eruptive variables, cataclysmic variables and X-ray variables. In pulsating variables, the stellar radii change with swelling and shrinking, which causes to change its spectrum. These pulses can be periodic, semi periodic or even irregular. When a star goes through irregular periods of variability caused by mass ejections and chromospheric activities, they are called eruptive variables. Cataclysmic variables are normally interacting binary systems that have white dwarfs, which go through large amplitude outbursts. X-ray variables are the binary system similar to cataclysmic variables, but having neutron stars or black holes.

1.1.3. Pulsating Variables

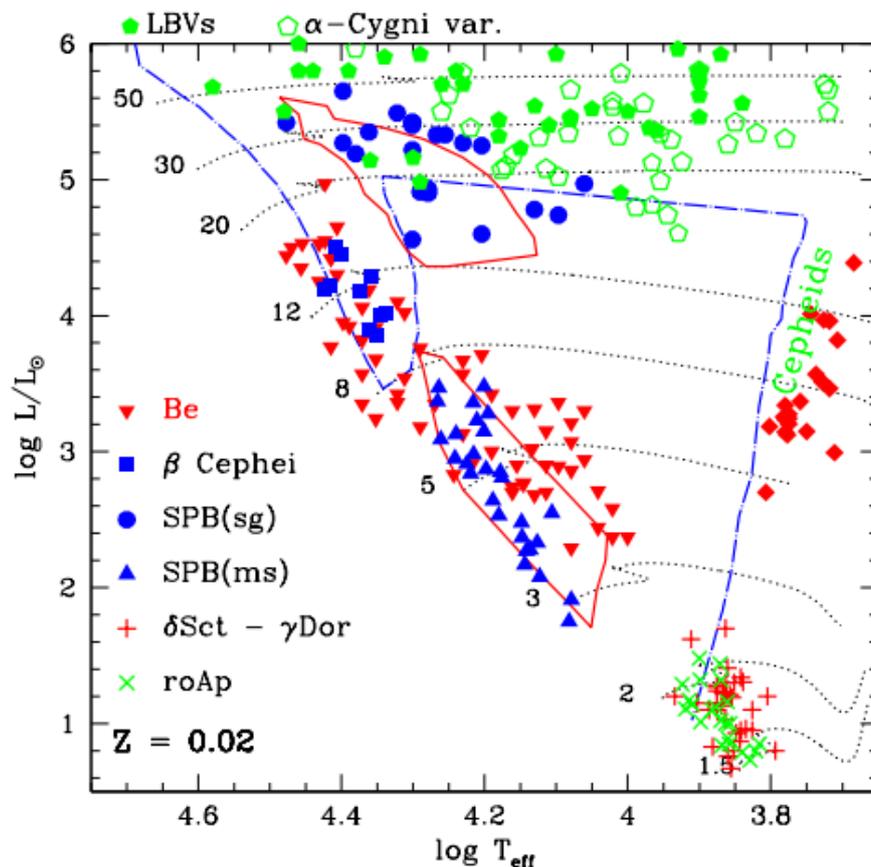


Fig. 1.2 H-R (Hertzsprung–Russell) diagram with classes of pulsating variable stars [04]

Every physical object has its' own natural frequencies of pulsation. There are numerous types of pulsating stars as shown in Fig. 1.2. When we consider a star, there are two main types of pulsations. The most obvious pulsation mode for stars is that it contains its spherical shape and changes its volume by a small amount. The matter which the star is formed move along with this changing of volume, which is in the radial direction. These types of variations called radial pulsations. These radial pulsations are similar to guitar string pulsations, which have fundamental, first overtones, second overtones and so on. However, in a star not like a guitar, these all modes can be occurred at the same time. The order (n) of the radial pulsations is normally defined in integer values as $n=0$, $n=2$ and $n=3$ and so on (see Fig 1.3). This is basically the different number of variable layers in the radial direction.

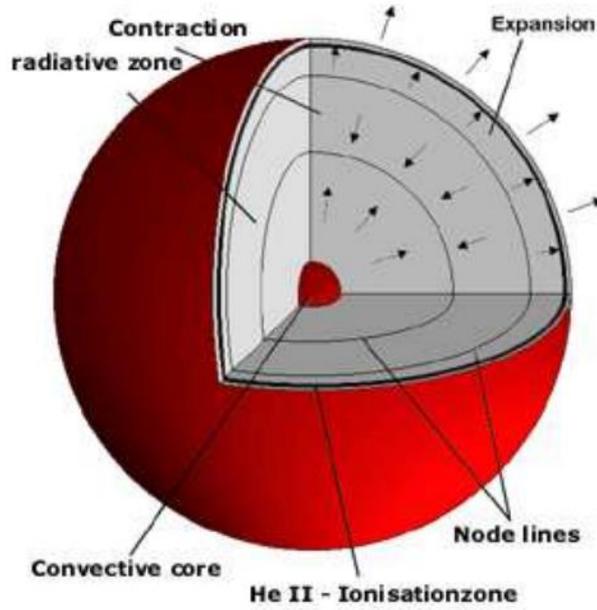


Fig. 1.3 Radial expansions of a pulsating variable star (order, $n=3$) [05]

The other type of pulsation is called non-radial pulsations. These pulsations occur in non-radial direction which is along the surface of the star. The star changes its shape, but keeps its volume constant. The number of nodes on the surface of the star denotes the non-radial modes. The total number of nodes are indicated by the degree 'l' and the number running through the pole is indicated by 'm' as given in the fig. 1.4.

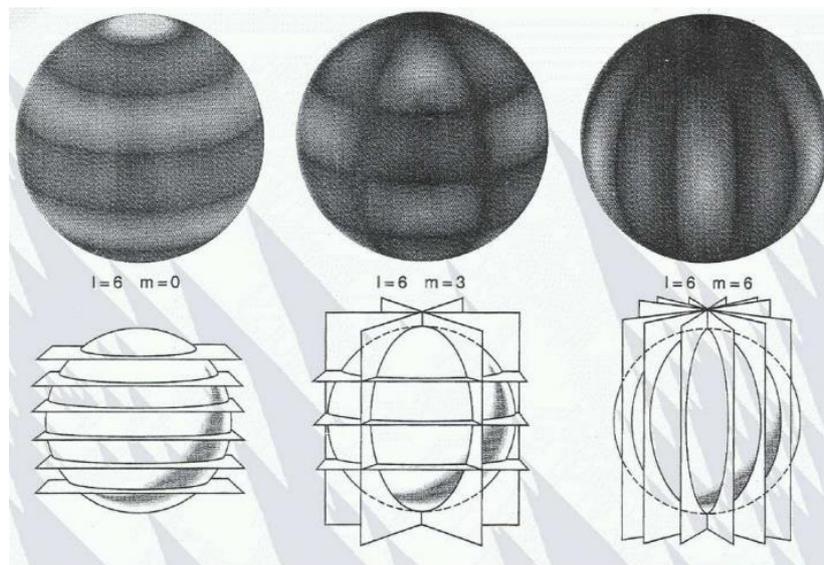


Fig. 1.4 Non-radial modes of a pulsating variable star for different l and m values [05]

Normally, the variable stars have these both pulsation modes. Consequently, a mode should be defined using spherical harmonic components n , l and m as shown in the fig. 1.5.

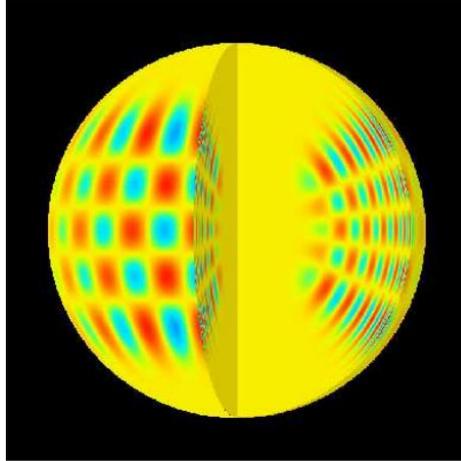


Fig. 1.5 High order (n) and high degree (l) pulsations. Different colors represent the surface cooling and heating [05]

1.2. Asteroseismology

Asteroseismology is one of the most important sectors of astronomy. Asteroseismology is the science that study the stellar oscillations. By analyzing these oscillations, the status of temperature, pressure and density of stellar interiors can be determined [06]. Generally, the frequency spectrum of a star has both gravitational modes and pressure modes with many harmonics and many combinations [07]. The resonance frequencies of the standing waves inside the star decides the oscillation pattern of the star. The speed of the wave mainly depends on the chemical composition and the temperature of the gas that the star formed of. By analyzing this wave pattern, intrinsic stellar properties such as gravity, temperature, chemical composition can be determined.

The field of Asteroseismology made a significant step with the invention of space telescopes such as Kepler and COROT. The observations of ground base telescopes were affected by weather, atmosphere and other numerous disturbances. However, space base telescopes can collect high quality, stable and reliable photometric data which made the asteroseismological calculations more accurate.

1.3. Kepler Mission

The Kepler Mission is launched in 2009 to search the Milky Way galaxy to discover the Earth-size and smaller planets (Extrasolar planets) in or near the habitable zone. Further, it aimed to find the fraction and the possibility of the hundreds of billions of stars in the Milky Way galaxy that might have such planets. Consequently, it led to discover more than 2,600 planets from outside our solar system, which could be promising places in search of life. These Earth like planets which rotates around the stars are designed to be detected using the transit method which is basically using the photometric data of the stars and discover the extrasolar planets by observing the eclipsing moments of those planets. This led Kepler mission to observe and collect the light curves of thousands of stars over nine years until 2018, its retirement. These photometric data provided new possibilities in various branches in Asteroseismology to analyze and learn more about the stars.

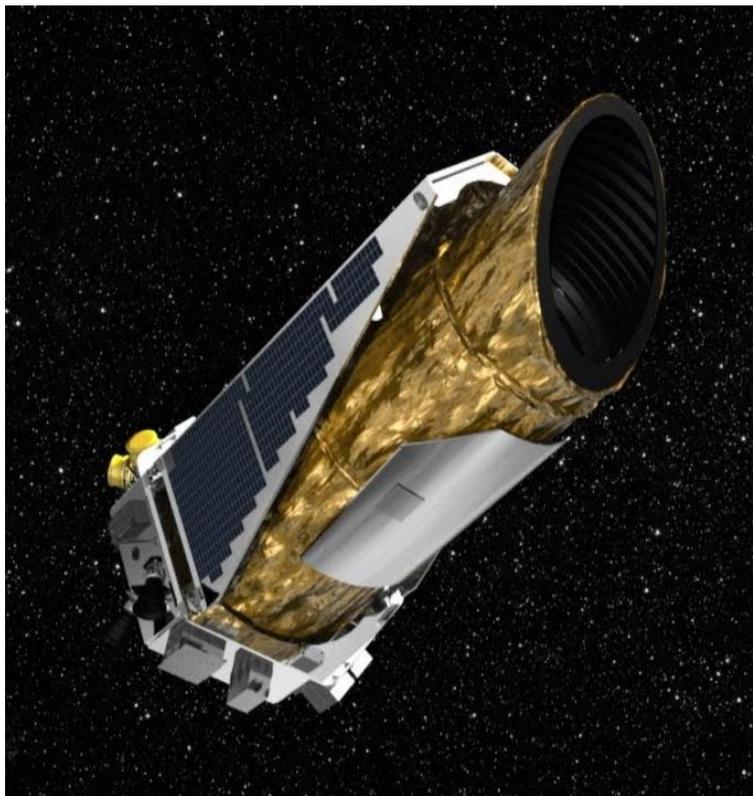


Fig. 1.6 Kepler space-based telescope [08]

Measuring the flux and its fluctuations with time of stellar objects is a main technique in asteroseismology that design to determine information and properties of stellar objects. These photometric systems have both ground base and space base telescopes to collect the photometric data to search for extrasolar planets and study stellar properties. Currently, with the development of the technology, scientists have developed better ways to obtain the photometric data, for example, Charged Coupled Devices (CCD). These techniques allowed to obtain more accurate observations compared to the photoelectric photometers. In addition, the CCDs can obtain observations from more than one object at once, unlike the photoelectric photometers. Scientists have developed computer-based applications to analyze these CCD observations and opened a gate to a new world of asteroseismology that seemed impossible few decades ago.

The objectives of this research project are to

1. Construct light curves from the Kepler Mission data
2. Obtain frequency values by analysing the time series
3. Identify the modes from the resulted frequencies and
4. Calculate basic internal properties using identified modes.

CHAPTER 2

LITERATURE REVIEW

2.1. Delta Scuti variable stars

The Delta Scuti (DSCT) variables are one of the main pulsation type variables along with RR Lyrae, RV Tauri, Mira class, Betelgeuse and with many other types of Cepheid variables. In fact, they are the most common pulsating variable among them. DSCT stars are pulsating variables of spectral types between A2 and F5 with luminosity classes V to III [09]. They lie on an extension of the Cepheid instability strip and their periods observed in the range of 0.02 days to 0.3 days (between about 30 minutes to 8 hours) [10]. They pulsate in radial and non-radial pulsation modes with photometric amplitudes less than 1 magnitude. After white dwarfs, they are the most common pulsing variables in the Milky Way galaxy.. DSCT stars have been particularly well studied because they can produce good prospects for asteroseismology. This is due to their many types of independent pulsation modes and also their high magnitude which leads it to be observed easily. Most of the independent pulsation modes in these stars are due to the κ mechanism in the He (Helium)II partial ionization zone with some contribution from H (Hydrogen) [11].

In this research, asteroseismic techniques were applied to three various DSCT type stars to obtain their intrinsic properties.

Table 2.1

Properties of the chosen DSCT variables that have been observed in the Kepler mission [12]

Star	Right Ascension	Declination	Kepler quarters	Data points
KIC 4048494	19h 16m 33.060s	+39 ⁰ 10' 28.70''	Q5.1	46157
			Q5.2	40433
KIC 4077032	19h 45m 03.175s	+39 ⁰ 11' 26.02''	Q2.1	41341
KIC 8623953	19h 25m 59.767s	+44 ⁰ 44' 45.31''	Q10.1	43978
			Q10.1	45006
			Q10.1	44061

2.1.1 Kappa Mechanism

DSCT stars have Helium rich atmosphere. The radial pulsations of these stars are caused by changing its radius by expanding and contracting around its equilibrium state. The Kappa Mechanism describes the star's expanding and shrinking phase and the fluctuations of luminosity is very important in this process. When the stellar gravity overcomes the radiation pressure, star contracts and its temperature rise. This causes atomic Helium to heat and then to become doubly ionized Helium (He^{2+}). The dimmest part of the variable star appears when the light blocked from escaping because He^{2+} is opaquer to the radiation. Subsequently, the increasing of stellar radiation pressure expands outer layers of the star against its gravity and the outer layers of the star become more transparent to the radiation as the star becomes more cooler and doubly ionized Helium becomes atomic Helium [13]. Therefore, the star becomes brighter and this cyclic process responsible for the periodic variation of the luminosity of the star. The structure and rotations of the DSCT stars are very complex and large number of modes are required to describe the oscillating behaviors observed in the light curves of these variable stars.

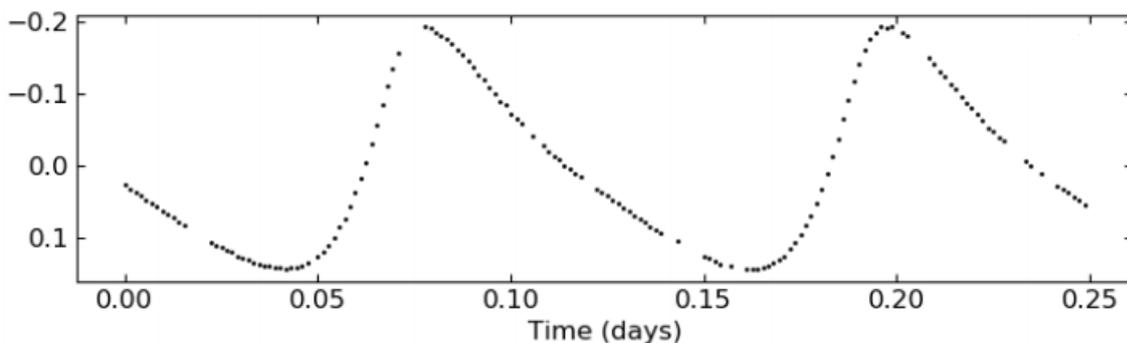


Fig. 2.7 Light curve of variable star SZ Lyn [14]

2.1.2. Previous Studies

Scientists have performed many research on delta Scuti stars from the early 1900s. In 1900 W.W. Campbell and W.H. Wright of Lick Observatory observed the radial velocity variations of star δ Scuti in the constellation Scautum by using the Mills spectrographs and This lead to the discovery of δ Scuti as a variable star [15]. In 1935 Edward Fath and Attilio Colacevich observed this object again using photometers and Mills spectrograph, and published a paper on their findings [16]. Due to their difference in average metallicity and period-luminosity

relationships with RR Lyrae stars, American astronomer Bradford Smith called these stars as Dwarf Cepheids. In 1969 M.S. Bessell introduced “AI Velorum stars” designation for all stars with amplitudes higher than 0.3 mag. In 1971 R. J. Dickens, V. A. French, P. W. Owst, A. J. Penny and A. L. T. Powell published a paper comparing metal abundance of Sun and these stars [17]. In 1979 Michel Breger suggested to call these variable stars “Delta Scuti stars” [18].

Because of their complexity of the rotation and the structure, scientists from all around the world have the enthusiasm to build up models to describe the internal dynamics of delta Scuti stars. To achieve their goals and get high quality data, terrestrial and space-based telescope networks have been constructed. Various research papers have been published using data from Kepler like campaigns. One of the significant research on the pulsation mechanism study of roAp star has been done by using Kepler photometric data and the similarity of pulsations to the DSCT stars and γ Doradus stars was by L. A. Balona in 2010 [19]. In 2015, S. Barceló Forteza completed another study of amplitude modulation in the DSCT star KIC 5892969 due to resonant mode coupling has been done using the observations of the Kepler mission [20].

The first DSCT star that considered in this research is KIC 4048494 which is also known as TYC 3121-984-1 [21].

Table 2.2
Basic properties of the chosen DSCT variable KIC 4048494

Name	KIC 4048494	
Right Ascension	19h 16m 33.060s	
Declination:	+39 ⁰ 10' 28.70''	
Parallax (mas):	1.06 ± 0.26 [22]	
Spectral type	A6V	
Fluxes [23]	B	9.747
	V	9.546
	R	9.504
	G	9.523

This star has been involved in some statistical research projects like ‘Finding binaries from phase modulation of pulsating stars with Kepler: V. Orbital parameters, with eccentricity and mass-ratio distributions of 341 new binaries’ by Simon J. Murphy [24].

The second DSCT star that considered is KIC 4077032 which is also known as HD 225644.

Table 2.3

Basic properties of the chosen DSCT variable KIC 4048494

Name	KIC 4048494	
Right Ascension	19h 45m 03.175s	
Declination:	+39 ⁰ 11' 26.02''	
Parallax (mas):	1.9789 ± 0.0389 [25]	
Spectral type	F2III _s	
Fluxes [26]	B	10.010
	V	9.71
	G	9.5674
	J	8.937

The third and last DSCT star that considered is KIC 8623953 which is also known as BD+443134.

Table 2.4

Basic properties of the chosen DSCT variable KIC 8623953

Name	KIC 8623953	
Right Ascension	19h 25m 59.767s	
Declination:	+44 ⁰ 44' 45.31''	
Parallax (mas):	2.7703 ± 0.0254 [27]	
Spectral type	A9V	
Fluxes [28]	B	9.55
	V	9.32
	G	9.3311
	J	8.874

All these stars have been involved in big data analysis like ‘The envelope of the power spectra of over a thousand δ Scuti stars. The $T_{\text{eff}} - v_{\text{max}}$ scaling relation’ By S. Barcelo forteza, T. Roca Cortes and R.A. Garcia [29].

CHAPTER 03

METHODOLOGY

3.1. Obtaining Target Pixel Files

First, the target pixel files were taken from the KASOC (<https://kasoc.phys.au.dk/>). For this research project, the researcher had to select only the short cadence data due to their large number of data points. The data set for the star KIC 4048494 was taken in the Quarter 5.2 and 5.2 in Kepler mission and they are dated 22nd of April to 23rd of June in 2010 which included 86590 data points. The data set for the second star KIC 4077032 was taken in the Quarter 2.1 and they were taken in 20th June to 20th July in 2009 which included 41341 data points. The third and the last star that considered to the project was KIC 8623953 and the data set of this tar was taken in Quarter 10 which dated 27th June to 28th September in 2011 and it included 133045 data points [30].

3.2. Null Data Reduction

Each of the obtained short cadence data files contains 40000+ points of one minute exposure data. All the data were taken according to the time variable and some of the data points corresponding to random time points contains null data sets which make errors in future calculations. Accordingly, these null data should be removed. This was done manually by checking each and every row in the data sets.

3.3. Plotting the Light Curves

After null data reduction, light curves must be plotted using the acquired data.

3.3.1. Plotting the Flux vs. Time Curves

After reduction of null data, remaining data were used to plot the Flux vs. Time Light Curves. This is done by using MATLAB computer based software. As the KASOC data were given in Kepler flux with corresponding time in days, these light curves were plotted without any calculations.

3.3.2. Acquiring the Polynomial fit

The polynomial fit was obtained using the ‘polyfit’ function in MATLAB software. When the Flux vs. Time light curves were plotted, this function can be used directly with the curve to obtain the polynomial fit. The degree of the polynomial can be adjusted with the function and the best fitting polynomial was selected.

3.3.1. Plotting the Brightness change (ppm) vs. Time Curves

Using the best polynomial fit for the curve and the Kepler flux to ppm conversion equation, the Brightness change (ppm) vs. Time (Days) can be plotted [y].

$$F_{\text{ppm}}(t) = 10^6 \left(\frac{F_{\text{kp}}(t)}{f(t)} - 1 \right) \quad (3.1)$$

Here,

F_{ppm} - Brightness change (ppm)

$F_{\text{kp}}(t)$ - Kepler flux

$f(t)$ - Best fitting polynomial

After calculating the Brightness change for each of the Kepler flux values. The brightness change vs. time light curve was plotted. With this procedure, the light curve becomes symmetric around y-axis (brightness change) and also the high values of Kepler flux (in the range of $10^6 - 10^7$) becomes less valued (in the range of $10^3 - 10^4$). Due to this the frequency analysis procedure becomes much easier to process.

3.4. Frequency Analysis using PERIOD04

With the modern robust analysis methods, the ability to calculate stellar properties such as mass, radius, luminosity, temperature, and composition have taken an immense step. Some of this information like Fourier analysis is very difficult to process in any other ways of analysis. In many cases, the nature of the variability is the one that provides the clues to the answers.

PERIOD04 is a frequency analyzing software that works on both Windows and Linux operating systems. It is a C++/Java based program that dedicated to statistical analysis of

large astronomical time series that have gaps and mainly the program includes tools to identify frequencies from large multiperiodic time series and provides an interface to perform frequency fits in a flexible way. General features of the PERIOD04 software are to operate discrete Fourier transformations, to calculate least-square fitting of multiple frequencies, functions to calculate uncertainties of the results and to calculate phase variations and amplitude variations.

PERIOD04 program has three basic modules and they all connect with the frequency analysis process. They are,

1. The Time String Module
2. The Fourier Module
3. The Fit Module

The time string module allows user to input the time string data. Within this module, it provides tools to combine data sets, split data into substrings, set weights, etc.

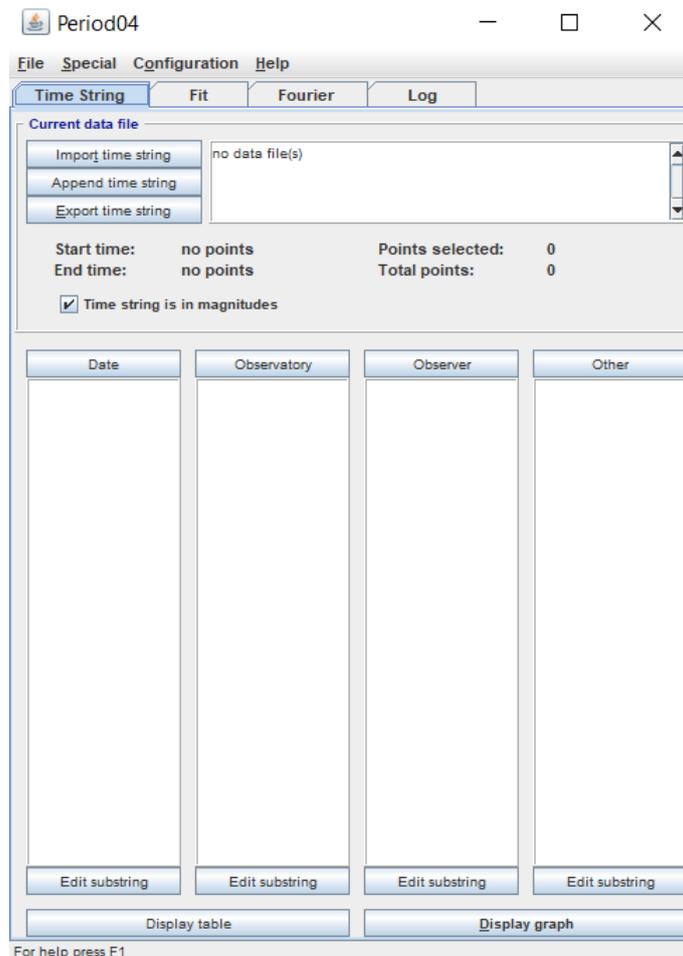


Fig. 3.1 The time string module of PERIOD04 program

The Fourier module is basically used for the extraction of new frequencies from the data. This process of frequency extraction is based on a discrete Fourier transform algorithm. There is an option called Fast Fourier transform (FFT) transform algorithm to extract frequencies, but since the time string data is not usually equally spaced, this method cannot be used.

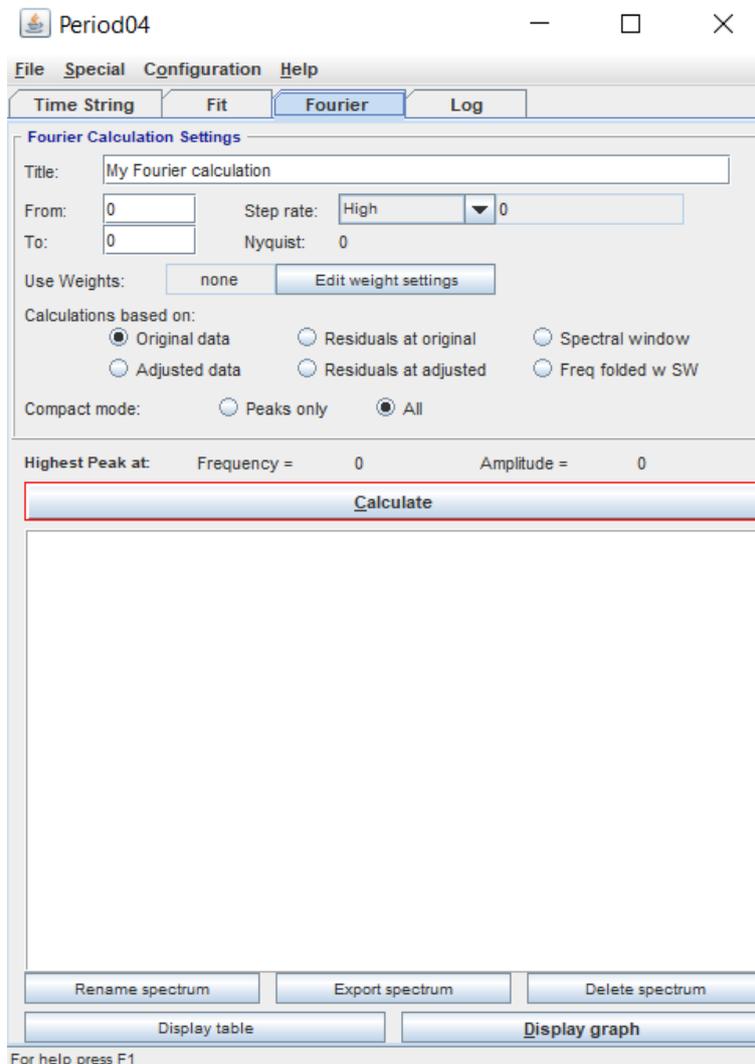


Fig. 3.2 The Fourier module of PERIOD04 program

The amplitude spectrum of a given star varies with time, t , and it can be expressed using the Fourier representation. The equation of this harmonic term is given in equation 3.2.

$$m = m_0 + \sum_{i=1}^N A_i \sin[2\pi(f_i t + \phi_i)] \quad (3.2)$$

Here,

- m_0 - zero-point
- A_i - amplitude of i^{th} term
- f_i - frequency of i^{th} term
- ϕ_i - phase of i^{th} term

In this Fourier analysis process, the program extracts the dominant frequencies from the amplitude spectrum and then calculate the corresponding amplitudes and their phases. The user can adjust the frequency step rate that the program works and smaller the step rate, results become more accurate. When a dominant frequency is found and calculated, before moving on to the next calculation, program subtract previously detected frequency. This is called the pre-whitening process. In this Fourier module, by ‘Display graph’ option, the residual light curves can be observed. When the program run for the first time, it calculates the peak frequency value and the ‘Display graph’ option shows a complete light curve of the given star. When the program run for the second time with pre-whitening process is on, it gives the second highest peak of the light curve. The residual light curve gives the same light curve to the previous step except the Highest peak of the light curve as shown in the Fig. 3.3.

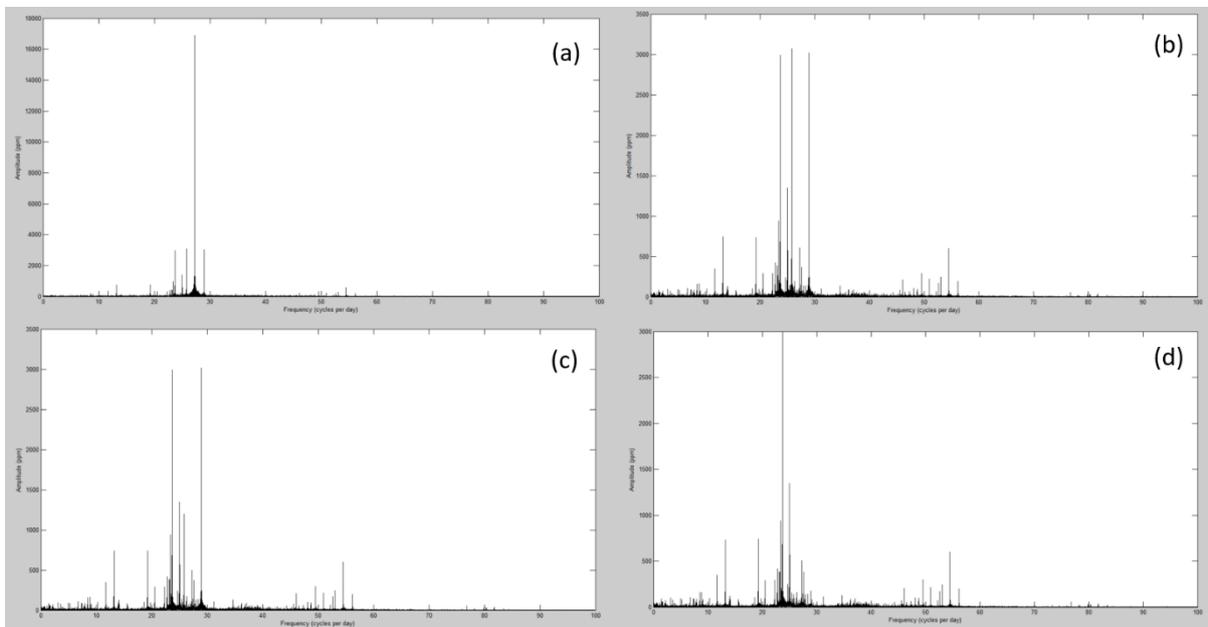


Fig. 3.3 Amplitude spectra of KIC 8623953.

The oscillation frequencies are the sharp peaks of each residual light curve. (a), (b), (c) and (d) show the remaining frequencies after the pre-whitening process

The fit module allows to fit the number of frequencies using the least-square method. PERIOD04 also have the tools to fit amplitudes, phase variations and also periodic time shifts. Besides, this module holds several tools like Monte Carlo simulations to calculate uncertainties for the fit parameters.

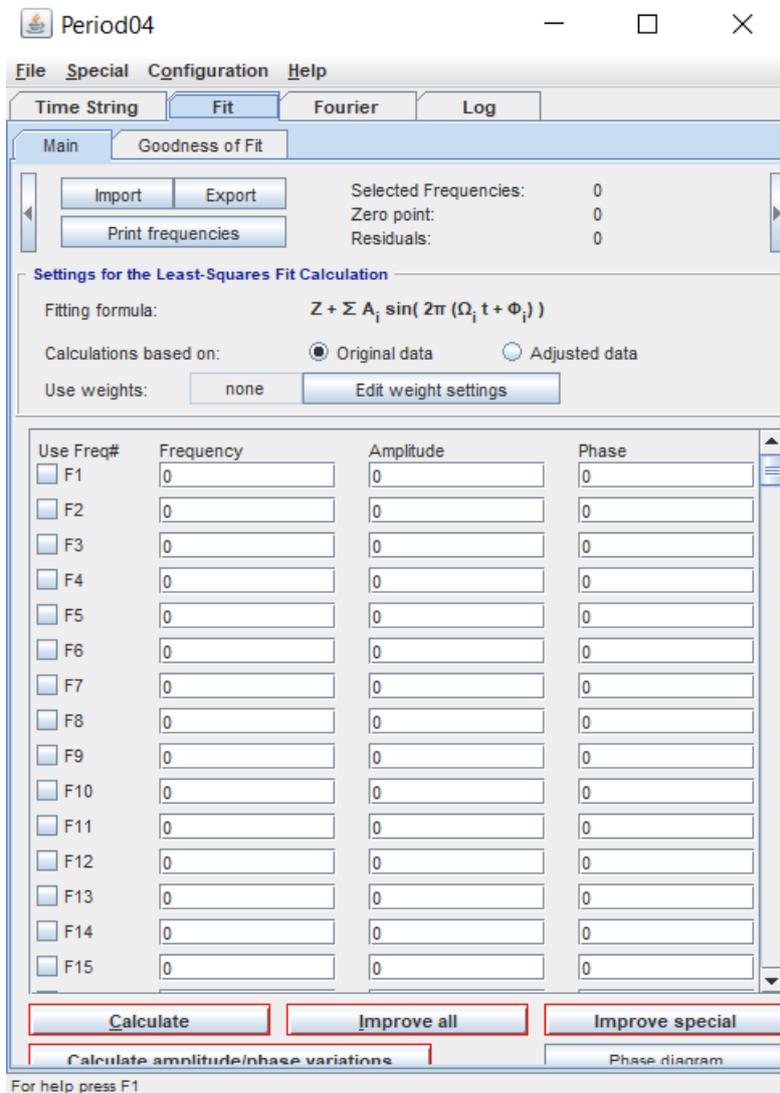


Fig. 3.2 The fit module of PERIOD04 program

3.5. Pulsation Mode Identification

After extracting the frequencies, the next most important step is to identify the frequency combinations and equidistant frequency patterns. These frequency combinations and overtones can be described in a straightforward manner using a general equation,

$$f = m f_1 \pm n f_2 \quad (3.3)$$

Here,

- f_1, f_2 - frequencies with highest amplitudes in the spectrum
 m, n - small integers

These f_1 and f_2 frequencies which can be used to express other frequencies are called the dominant frequencies. In the radial mode identifying process, the period ratio of frequencies is a good approximation.

For DSCT stars,

$$0.756 < \frac{P_2}{P_1} < 0.786 \quad - \text{ Then } P_1 \text{ is the fundamental radial and } P_2 \text{ is the 1}^{\text{st}} \text{ overtone}$$

$$0.611 < \frac{P_3}{P_1} < 0.632 \quad - \text{ Then } P_1 \text{ is the fundamental radial and } P_3 \text{ is the 2}^{\text{nd}} \text{ overtone}$$

But also if the amplitude of a frequency is sufficiently large, then this frequency can be considered as the fundamental radial.

3.6. Pulsation Constant

Pulsation constant is defined as,

$$Q = P_1 \sqrt{\frac{\rho}{\rho_{\odot}}} \quad (3.4)$$

Here,

- P_1 - fundamental pulsation period in days
 ρ - mean stellar density

Building a relationship to pulsation period and mean stellar density with known variables of the star, Q value can be determined.

According to the Stefan - Boltzmann law,

$$\frac{L}{L_{\odot}} = \left(\frac{R}{R_{\odot}}\right)^2 \left(\frac{T_{eff}}{T_{eff,\odot}}\right)^4 \quad (3.5)$$

Here,

L - luminosity of the star

T_{eff} - effective temperature of the star

R - radius of the star

A relation to the mean density of the star with respect to surface gravity(g) and mass(M) can be given as,

$$\rho \propto \frac{M}{R^3} \propto \frac{g}{R} \quad (3.6)$$

Furthermore, the relationship between the luminosity of the star and the bolometric magnitude (M_{bol}) can be given as,

$$M_{bol} - M_{bol,\odot} = -2.5 \log\left(\frac{L}{L_{\odot}}\right) \quad (3.7)$$

From equations (3.4), (3.5), (3.6) and (3.7), the relationship between Q and know parameters can be express as,

$$\log Q = -6.454 + \log P + 0.5 \log g + \log T_{eff} + 0.1 M_{bol} \quad (3.8)$$

Using this equation (3.8), the Q value corresponding to each frequency can be calculated.

CHAPTER 04

RESULTS AND DISCUSSION

4.1. KIC 4048494

The following are the findings related to the star KIC 4048494.

4.1.1. Flux vs. Time Curve

After proceeding the null data reduction process, the Light curves were plotted using direct readings (Fig. 4.1 and Fig 4.2).

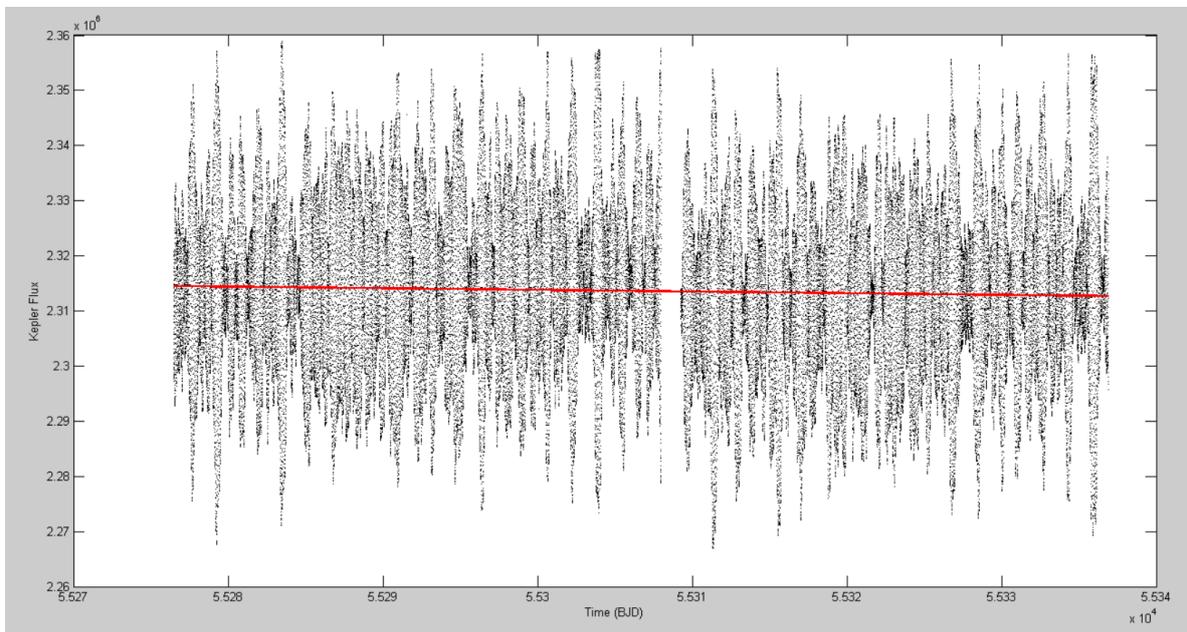


Fig. 4.1 Flux vs. Time curve for KIC 4048494 and the red line indicates the best fitting polynomial (1st order) for the fit

4.1.2. Brightness change (ppm) vs. Time Curves

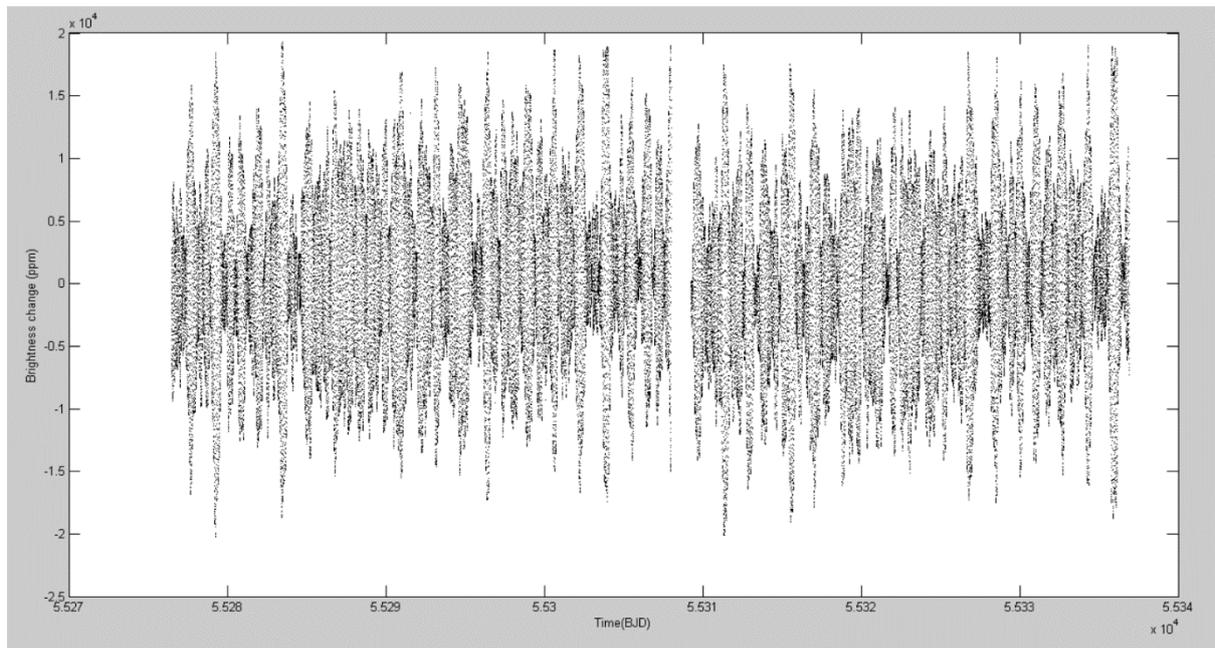


Fig. 4.2 Brightness change (ppm) vs. Time curve for KIC 4048494

4.1.3. Frequency analysis

(Calculations Done fo the Q values will be added to the appendix.)

Table 4.1

Frequencies of DSCT variable KIC 4048494 and their Identifications

Frequency (cycles/day)	Frequency (μHz)	Amplitude (ppm)	Phase (rad)	Q value (days)	Identification
14.363	166.244	5191.116	0.092	0.0201	f1
15.581	180.341	4967.593	0.903	0.0186	f2
14.894	172.383	3911.696	0.881	0.0194	f3
13.709	158.668	3358.444	0.491	0.0211	f4
15.140	175.227	1198.554	0.319	0.0191	f5
11.058	127.992	1434.138	0.955	0.0262	f6
0.027	0.316	844.397	0.303	10.5985	f7
13.919	161.101	556.754	0.259	0.0208	f8
12.758	147.664	537.456	0.171	0.0227	f9
13.394	155.028	487.141	0.365	0.0216	f10
13.433	155.479	1397.078	0.696	0.0215	f10+f7
29.946	346.594	526.655	0.307	0.0097	f1+f2
15.861	183.578	503.556	0.115	0.0182	2f3+f8
15.793	182.793	430.497	0.996	0.0183	2f10-f6
15.625	180.849	361.842	0.954	0.0185	f2+2f7
0.051	0.594	392.378	0.944	5.6407	f9-f8
12.809	148.257	329.061	0.224	0.0226	f9+2f7
15.663	181.280	328.252	0.320	0.0185	f2+3f7
15.231	176.281	262.369	0.868	0.0190	f5+3f7
27.665	320.199	240.730	0.179	0.0105	f8+f4+f7
0.077	0.891	225.540	0.199	3.7605	f5-f2
12.149	140.615	208.879	0.417	0.0238	2f9-f10
15.140	175.237	875.112	0.562	0.0191	
14.576	168.705	791.383	0.150	0.0199	
16.536	191.394	530.530	0.794	0.0175	

4.1.4. Amplitude (ppm) vs. Frequency Curve

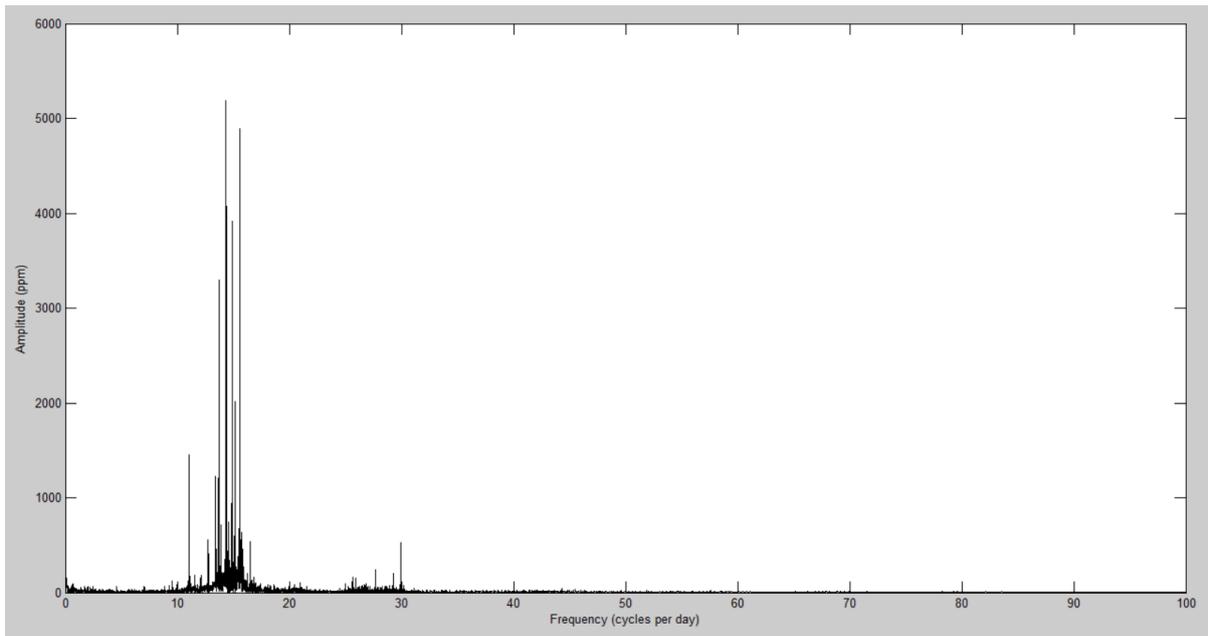


Fig. 4.3 Amplitude (ppm) vs. Frequency curve for KIC 4048494

4.2. KIC 4077032

The following are the findings related to the star KIC 4077032.

4.2.1. Flux vs. Time Curve

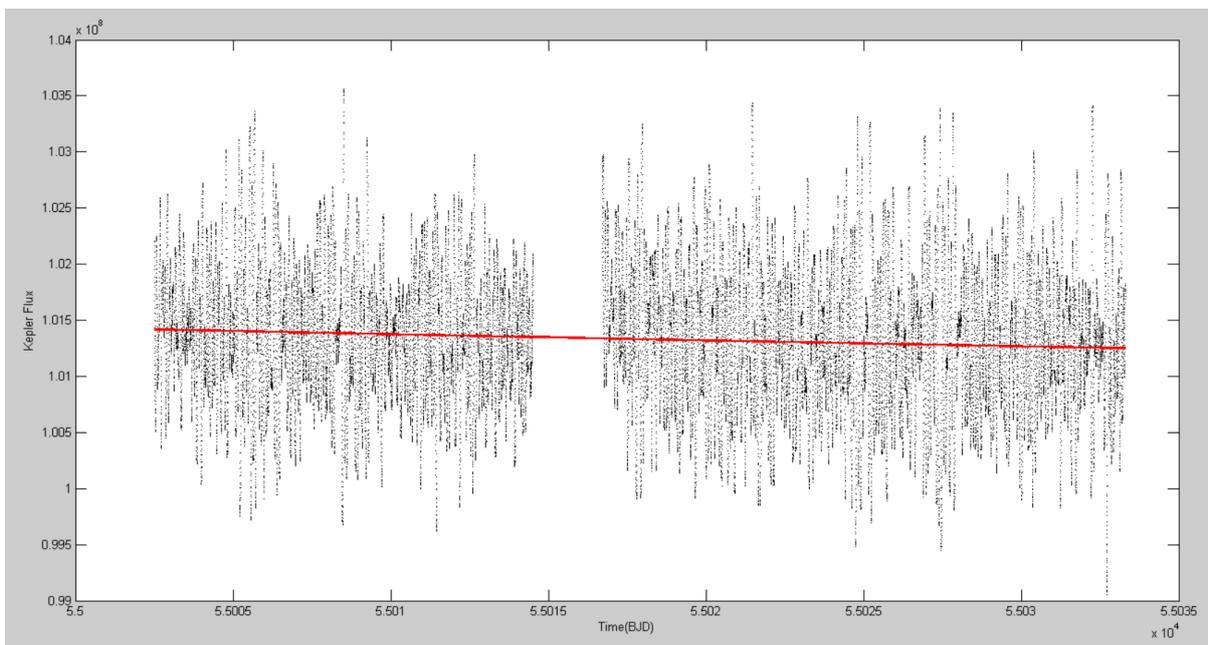


Fig. 4.4 Flux vs. Time curve for KIC 4077032 and the red line indicates the best fitting polynomial (2nd order) for the fit

4.2.2. Brightness change (ppm) vs. Time Curves

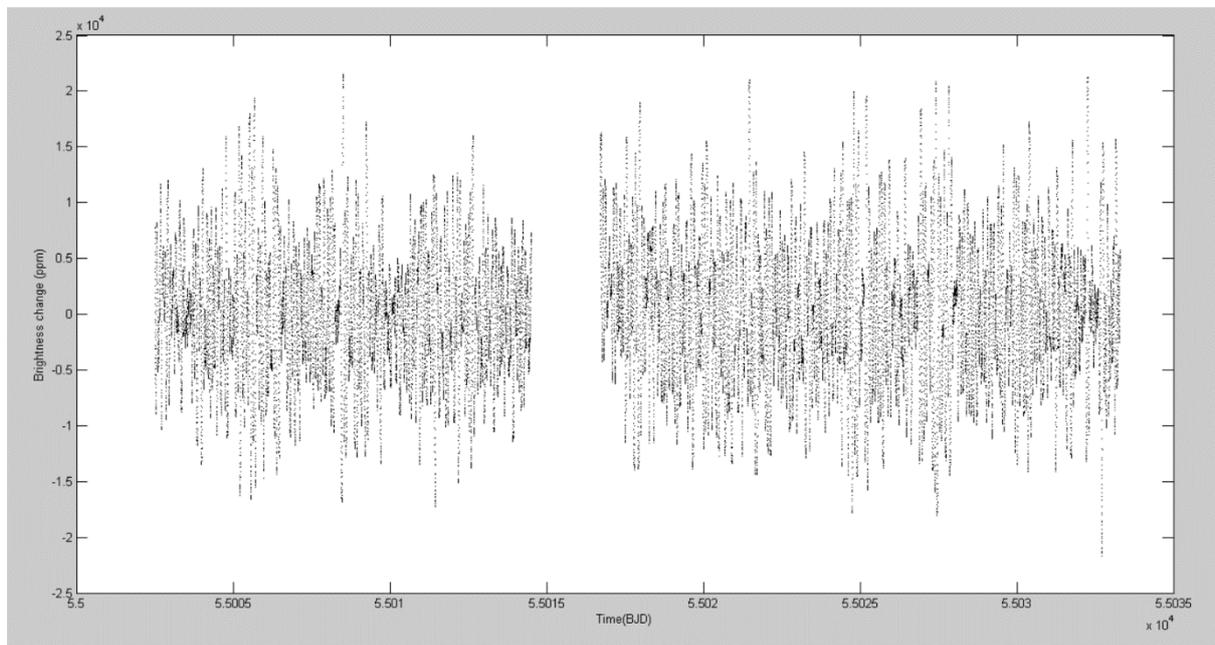


Fig. 4.5 Brightness change (ppm) vs. Time curve for KIC 4077032

4.2.3. Frequency analysis

Table 4.2

Frequencies of DSCT variable KIC 4077032 and their Identifications

Frequency (cycles/day)	Frequency (μHz)	Amplitude (ppm)	Phase (rad)	Q value (days)	Identification
14.482	167.615	5064.259	0.686	0.0232	f1
8.037	93.025	2965.640	0.618	0.0418	f2
9.640	111.574	1765.329	0.222	0.0349	f3
12.189	141.079	6049.985	0.626	0.0276	f4
6.311	73.048	1343.633	0.151	0.0532	f5
6.706	77.615	713.238	0.743	0.0501	f6
0.070	0.808	525.519	0.972	4.8132	f7
14.480	167.596	2256.741	0.334	0.0232	3f2-f3
12.046	139.425	3098.255	0.089	0.0279	3f6-f2
9.736	112.683	1425.470	0.664	0.0345	f3+f7
16.299	188.644	1423.263	0.501	0.0206	3f3-2f5
14.626	169.288	890.005	0.787	0.0230	f1-f2-f5
12.334	142.752	994.331	0.760	0.0272	f1-f2+f4-f5
7.412	85.790	755.948	0.876	0.0453	3f6-2f5-f7
9.380	108.567	777.302	0.181	0.0358	2f2-f6
13.430	155.437	702.408	0.904	0.0250	2f6
12.246	141.737	2463.932	0.409	0.0274	f4+f7
12.598	145.815	647.719	0.565	0.0267	2f5
15.984	184.999	520.241	0.559	0.0210	f3+f5
10.036	116.160	506.610	0.433	0.0335	2f1-3f5
14.743	170.641	522.780	0.351	0.0228	f6+f2
14.799	171.280	2894.418	0.593	0.0227	
12.152	140.647	1782.035	0.381	0.0277	
14.058	162.710	1652.115	0.717	0.0239	

4.2.4. Amplitude (ppm) vs. Frequency Curve

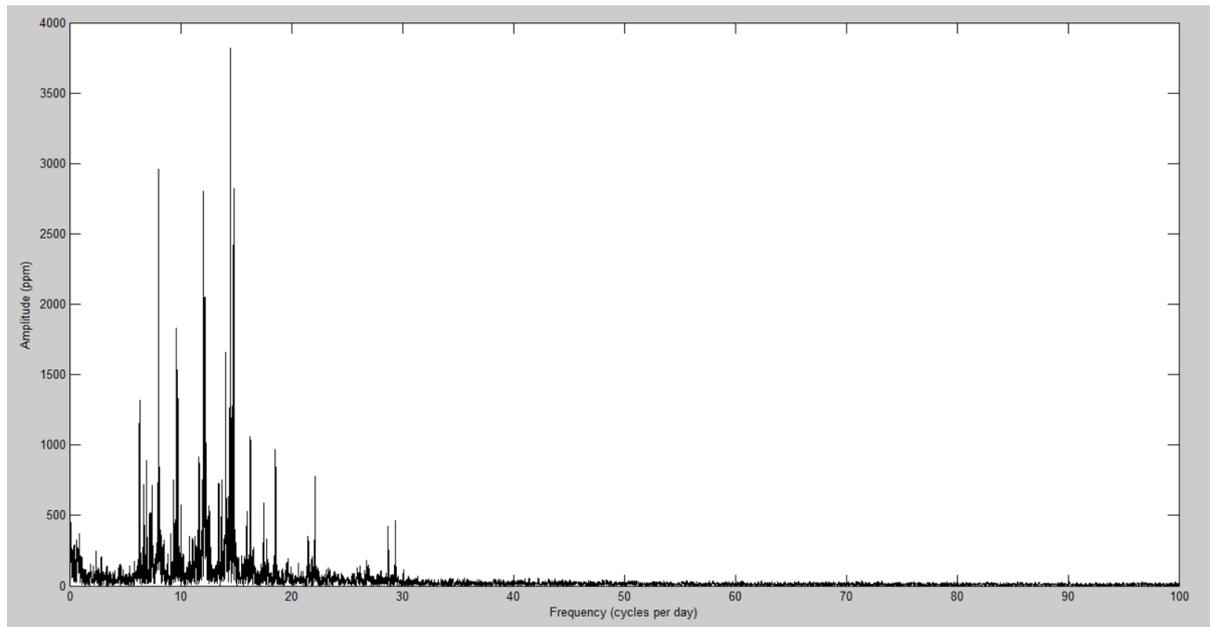


Fig. 4.6 Amplitude (ppm) vs. Frequency curve for KIC 4077032

4.3. KIC 8623953

The following are the findings related to the star KIC 8623953.

4.3.1. Flux vs. Time Curve

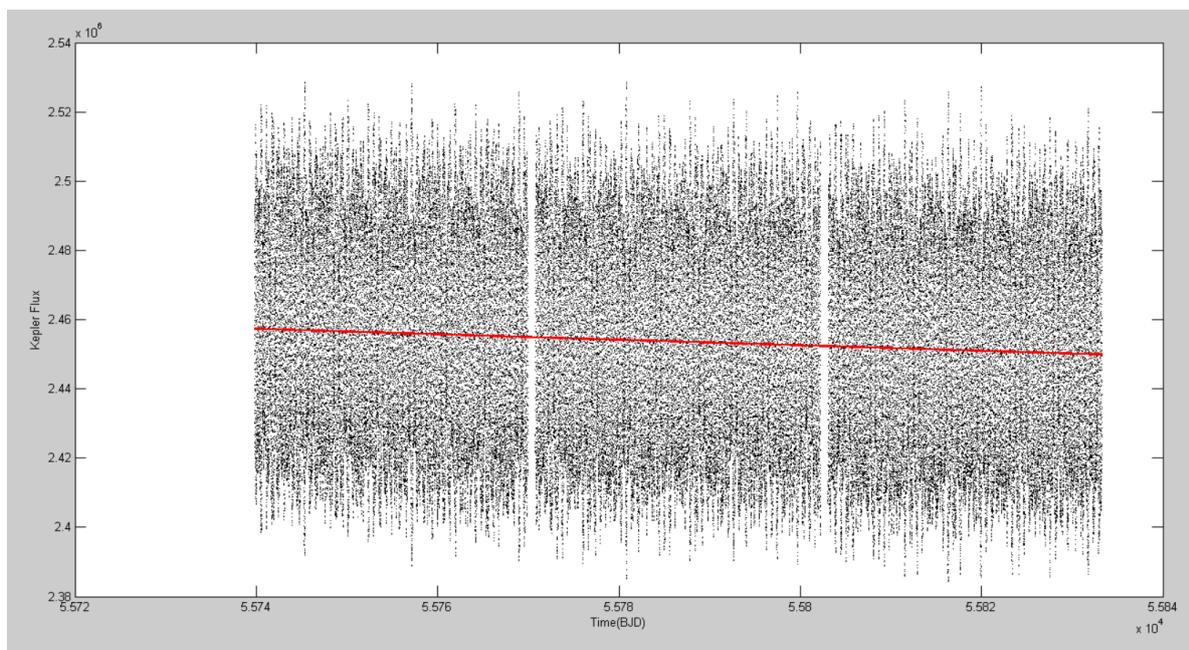


Fig. 4.7 Flux vs. Time curve for KIC 8623953 and the red line indicates the best fitting polynomial (1st order) for the fit

4.3.2. Brightness change (ppm) vs. Time Curves

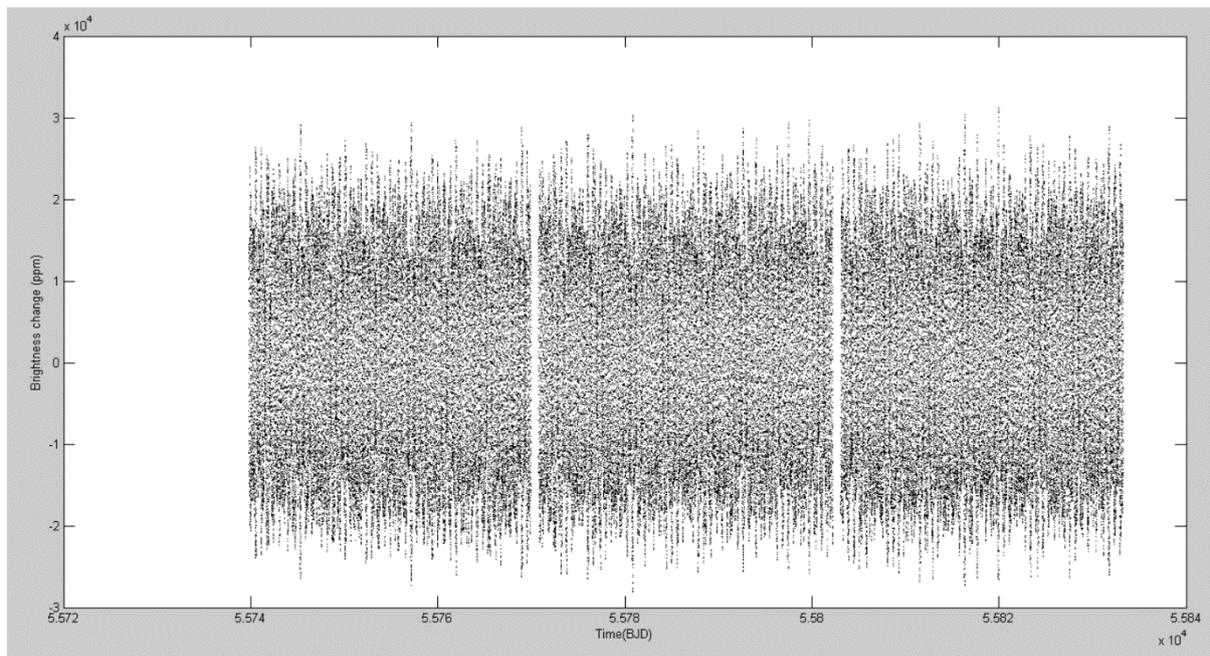


Fig. 4.8 Brightness change (ppm) vs. Time curve for KIC 8623953

4.3.3. Frequency analysis

Table 4.3

Frequencies of DSCT variable KIC 8623953 and their Identifications

Frequency (cycles/day)	Frequency (μHz)	Amplitude (ppm)	Phase (rad)	Q value (days)	Identification
27.257	315.473	16604.431	0.806	0.0105	f1
25.821	298.857	3061.353	0.295	0.0111	f2
23.706	274.369	3007.025	0.021	0.0121	f3
23.426	271.137	966.719	0.458	0.0122	f4
13.205	152.841	1033.020	0.250	0.0217	f5
11.740	135.884	489.412	0.651	0.0244	f6
0.022	0.254	224.790	0.153	13.0486	f7
28.946	335.025	3019.627	0.557	0.0099	3f1-4f5
24.976	289.078	1347.585	0.758	0.0115	f5+f6
19.253	222.836	751.530	0.764	0.0149	2f1-3f6-f7
54.514	630.953	599.807	0.310	0.0053	2f1
22.781	263.674	417.286	0.988	0.0126	f2+f4-2f5
23.194	268.449	387.507	0.895	0.0123	2f4-f3
49.527	573.233	295.519	0.925	0.0058	f2+f3
24.635	285.127	257.616	0.053	0.0116	3f2-4f5
27.275	315.678	268.360	0.248	0.0105	f1+f7
53.079	614.337	246.843	0.136	0.0054	f1+f2
50.962	589.843	206.418	0.733	0.0056	f1+f3
13.205	152.835	326.583	0.381	0.0217	
25.094	290.434	555.476	0.845	0.0114	
27.264	315.560	618.572	0.041	0.0105	
27.604	319.487	369.328	0.945	0.0104	
11.740	135.884	278.462	0.933	0.0244	
22.284	257.914	305.149	0.012	0.0128	

4.3.4. Amplitude (ppm) vs. Frequency Curve

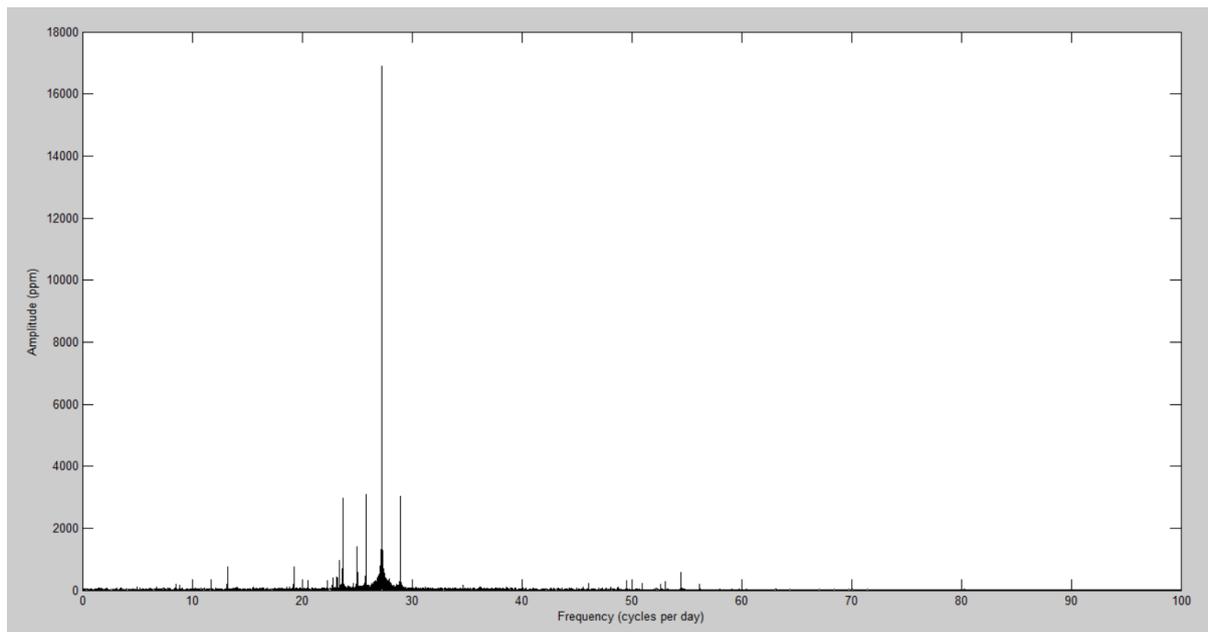


Fig. 4.9 Amplitude (ppm) vs. Frequency curve for KIC 8623953

4.4. Discussion

4.4.1. Light curves

The Kepler flux vs. time curve was plotted for each star after doing the null data reduction. The best fitting curve of this curve is the symmetric line to the sinusoidal curve. However, unlike in a normal sinusoidal curve, this light curve does not occur around x-axis (Time). In fact, it situated around a positive y value (positive flux value) of the graph. Further, this symmetric line which we found as the best fitting polynomial is not perfectly horizontal and even it even might not be a line.

In the DSCT star KIC 4048494, there were three Kepler quarter short cadence data files (Q5.1, Q5.2 and Q5.3). After plotting the flux vs. time curve, it was clear that the curves' symmetry was disrupted by a large amount. In consequence, the best fitting polynomial becomes a large order polynomial, and even that was not able to fit the curve as shown in Fig. 4.7. This can happen due to the rotation of the Kepler telescope because of its' three month 90° adjusting procedure. To avoid this error, the non-aligning short cadence file Q5.1 was not included in the research. Even this file was not considered, there were 86590 time series data points to continue the research for the particular star.

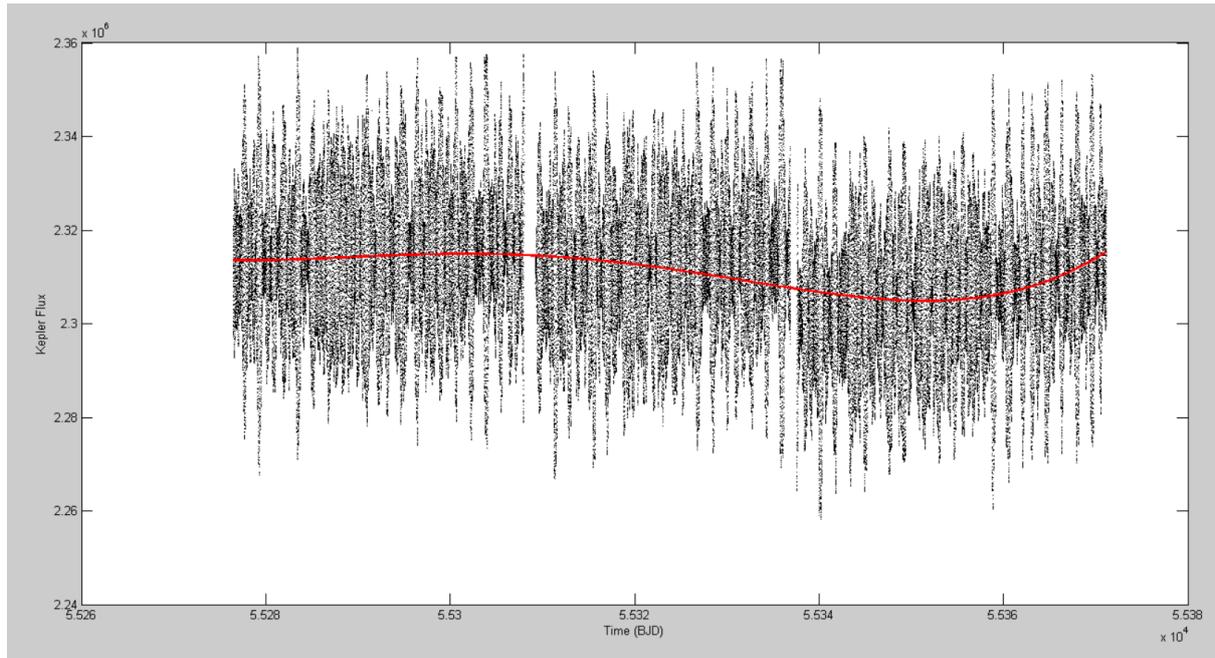


Fig. 4.10 Flux vs. Time curve for KIC 86590 with all the three data files (Q5.1, Q5.2 and Q5.3). The best fitting polynomial is in the order 5.

The best fitting curve of the star KIC 4048494 was a 1st order polynomial, for the second star KIC 4077032 it was a 2nd order polynomial and for the third star KIC 8623953, it was a 1st order polynomial. Using the Kepler flux to ppm conversion equation (equation 3.1), this unsymmetric light curves were converted into symmetric curves; which were shown as the brightness change (ppm) vs. time curves. This resulted time series with ppm values can be directly used to proceed the frequency analysis step.

4.4.1. Frequency Analysis

In the frequency analysis progress in the PERIOD04 program, the frequency range was set 0 to 100 cycles per day to include all the possibilities. The frequencies of DSCT type variables normally belong to 0 to 50 cycles per day and rarely they have frequencies over 50 cycles per day. Step rates were set to high to get the most accurate results.

Table 4.4

Used step rates in PERIOD04 program for each star

Star	Step Rate ($\times 10^{-3}$ cycles/days)
KIC 4048494	0.827
KIC 4077032	1.623
KIC 8623953	0.535

The detected frequencies were identified as linear combinations of other fundamental frequencies (f_1 and f_2). These linear combinations can be helpful to identify the radial modes of the star. This cannot be considered when we calculate the non-radial pulsations.

Table 4.5

Period ratio calculation for KIC 4048494 and KIC 4077032

Star	f_1 (μHz)	P_1 (days)	f_2 (μHz)	P_2 (days)	$\frac{P_2}{P_1}$
KIC 4048494	127.992	0.090	166.244	0.070	0.770
KIC 4077032	73.048	0.158	93.025	0.124	0.785

Because of these period ratio values, for KIC 4048494, the fundamental radial mode is 127.992 μHz and the first overtone is 166.244 μHz . The fundamental radial mode of KIC 4077032 is 73.048 μHz and the first overtone is 93.025 μHz .

The highest amplitude of the KIC 8623953 is very large compared to the other amplitudes. As a result, the star's fundamental radial should equal its highest frequency, which is 315.473 μHz .

CHAPTER 5

CONCLUSIONS AND FUTURE RESEARCH

The purpose of this research was to construct light curves from the Kepler Mission data and calculate basic internal properties of the selected stars by analysing the modes of the stars. Accordingly, the researcher first constructed the light curves using Kepler data and the frequency values were determined by using that data. Then from the frequency results, the radial modes and the pulsating constants were calculated.

More than 60 days of continuous short cadence Kepler observations were used to detect 25 independent frequencies for KIC 4048494 and 30 days of observations were used to obtain 24 independent frequencies for KIC 4077032. Continuous observations of 90 days were used to obtain 25 independent frequencies for KIC 8623953. These KIC 4048494, KIC 4077032 and KIC 8623953 DSCT stars were selected based on the fact that the DSCT stars contain many frequencies in their spectra and also their complex nature of frequency combinations. These frequency combinations were identified using simple linear combinations. Also, the high luminosity values make DSCT stars to observe more accurately. The radial fundamental mode and the 1st overtone of each star was determined by observing their period ratios and considering their amplitude variations. For the star KIC 4048494, the fundamental radial mode was 127.992 μHz and the first overtone was 166.244 μHz . The fundamental radial mode for the star KIC 4077032 was 73.048 μHz and the first overtone was 93.025 μHz . The fundamental radial frequency of the star KIC 8623953 was 315.473 μHz .

From the frequency analysis process, it is clear that the highest peaked frequency is not always become the rotational fundamental mode. Both KIC 4048494 and KIC 4077032 do not have the highest Amplitude frequency as the rotational fundamental mode. However, in KIC 8623953 the Amplitude difference between the peaked frequency is very large compared to the rest of the frequencies and due to that the rotational fundamental frequency can be consider as the peaked frequency. This frequency analysis process through PERIOD04 program had do continue work one by one and with the high step configuration and pre-whitening process is on, the process took some considerable amount of time.

The next step in this research was to obtain the non-radial modes. This can proceed with studying the radial modes and frequency combinations and finally extracting the ' l ' values.

For an example, if we can identify quintuplet in the frequency table, the l value becomes 2 and from that, the 'm' values can be found ($m = 2l+1$). When we know the n , l and m values of a star, the stellar rotation period can be found. Moreover, using the fundamental pulsation period values, mean density of the star can be obtained.

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APPENDIX

Calculations done for calculating Q values.

For KIC 4048494

freq.	Period	Parallax	d	m	M	BC	M bol.	log(g)	T eff.	Q
14.363	0.070	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.020
15.581	0.064	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.019
14.894	0.067	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.019
13.709	0.073	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.021
15.140	0.066	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.019
11.058	0.090	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.026
0.027	36.623	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	10.599
13.919	0.072	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.021
12.758	0.078	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.023
13.394	0.075	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.022
13.433	0.074	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.022
29.946	0.033	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.010
15.861	0.063	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.018
15.793	0.063	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.018
15.625	0.064	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.019
0.051	19.492	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	5.641
12.809	0.078	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.023
15.663	0.064	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.018
15.231	0.066	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.019
27.665	0.036	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.010
0.077	12.995	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	3.761
12.149	0.082	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.024
15.140	0.066	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.019
14.576	0.069	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.020
16.536	0.060	1.610	621.22	9.546	0.580	0	0.580	3.951	7621	0.018

For KIC 4077032

freq.	Period	Parallax	d	m	M	BC	M bol.	log(g)	T eff.	Q
14.482	0.069	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.023
8.037	0.124	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.042
9.640	0.104	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.035
12.189	0.082	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.028
6.311	0.158	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.053

6.706	0.149	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.050
0.070	14.323	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	4.813
14.480	0.069	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.023
12.046	0.083	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.028
9.736	0.103	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.035
16.299	0.061	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.021
14.626	0.068	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.023
12.334	0.081	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.027
7.412	0.135	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.045
9.380	0.107	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.036
13.430	0.074	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.025
12.246	0.082	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.027
12.598	0.079	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.027
15.984	0.063	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.021
10.036	0.100	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.033
14.743	0.068	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.023
14.799	0.068	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.023
12.152	0.082	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.028
14.058	0.071	2.033	491.807	9.71	1.251	0	1.251	4.047	6789	0.024

For KIC 8623953

Freq.	Period	Parallax	d	m	M	BC	M bol.	log(g)	T eff.	Q
27.257	0.037	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.011
25.821	0.039	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.011
23.706	0.042	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.012
23.426	0.043	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.012
13.205	0.076	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.022
11.740	0.085	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.024
0.022	45.578	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	13.049
28.946	0.035	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.010
24.976	0.040	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.011
19.253	0.052	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.015
54.514	0.018	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.005
22.781	0.044	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.013
23.194	0.043	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.012
49.527	0.020	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.006
24.635	0.041	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.012
27.275	0.037	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.010
53.079	0.019	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.005
50.962	0.020	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.006
13.205	0.076	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.022

25.094	0.040	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.011
27.264	0.037	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.011
27.604	0.036	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.010
11.740	0.085	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.024
22.284	0.045	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.013
20.514	0.049	2.779	359.893	9.32	1.539	0	1.539	3.738	7725	0.014